3.01 FORCES AND PRESSURE.

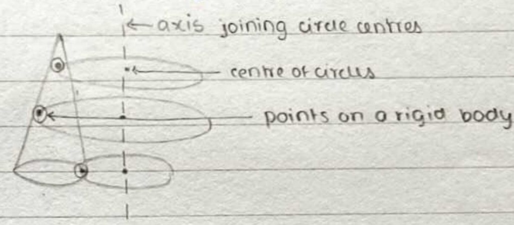
FORCES AND TURNING EFFECTS

Axis

Any turning effect on a body must take place about an axis (Rotations take place about an axis)

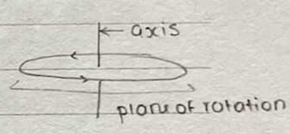
Definition: the line joining all the centre of circles described by the points on a rigid body.

> An axis is a straight line when involving a rigid body and not a fluid

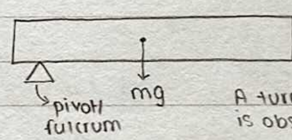


Turning effect of force

- The axis of rotation is through the fulcrum, perpendicular to the plane of rotation
- Anything balanced about the centre of mass produces no net turning effect.



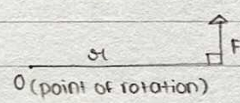
- aka torque, $\vec{\Gamma}$
  aka moment of force
- S.I unit : Nm ; it is a vector quantity.



A turning effect of force is observed. Force here: mg

$\vec{\Gamma} = \bar{r} \times \bar{F}$ (if $\bar{r} \perp \bar{F}$)

$\Gamma = F \times r \sin\theta$



> clockwise / anticlockwise torque
> CW / ACW moments = CWM / ACWM

Principle of moments / rotational equilibrium

For a body to be balanced the total sum of the CWM is equal to the total sum of the ACWM

(Balanced when net ACWM = net CWM)

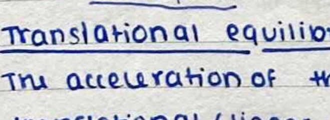
If a body is at equilibrium, the total sum of the clockwise moments about any point is equal to the sum of the anticlockwise moments about the same point.

> At the moment when the system just begins to rotate (time = 0) the upward forces = the downward forces.

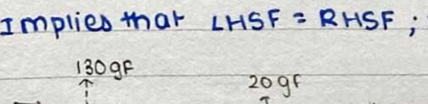
Translational equilibrium

The acceleration of the entire system taken as a whole = 0, then the system is under translational / linear equilibrium.

Implies that LHSF = RHSF ; but it isn't under rotational equilibrium.



$(100 + 50) - (20) = 130$ gF ↑



Downwards F = 20 + 60 + 50 = 130 N

Upwards F : 100 + 30 = 130 N

Upwards F = Downwards F so body is at equilibrium.

[in answers write "according to principle of moments ACWM = CWM (as body is at equilibrium)"]

Torque produced by any force

(i) acting on the point of rotation $= 0$

(ii) parallel to $r = 0$

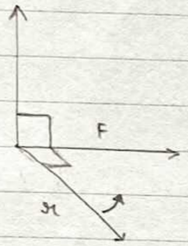
If r is increased more torque is generated.

F is parallel to r. $\bar{\tau} = 0$

Direction of torque

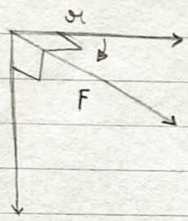
- perpendicular to the plain of the paper , doesn't lie in the plain of $\bar{r}$ and $\bar{F}$'s directions.
  (⊥ to r and F)

(Rotate r on F)

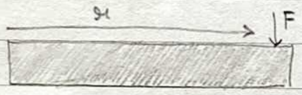
① 

anticlockwise ⇒ outwards ⇒ upwards
(when a screw is moved this way it moves up)

• : out : ACWM
x : in : CWM
inward vector ⊥ to plain of paper

② 

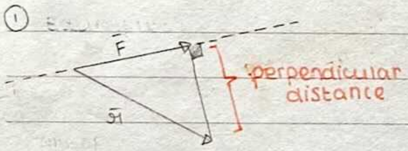
clockwise ⇒ inwords ⇒ downwards

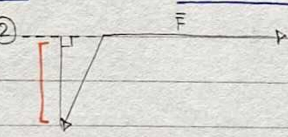


- point of origin to be same
- parallel shifting of vectors

Stability and equilibrium.

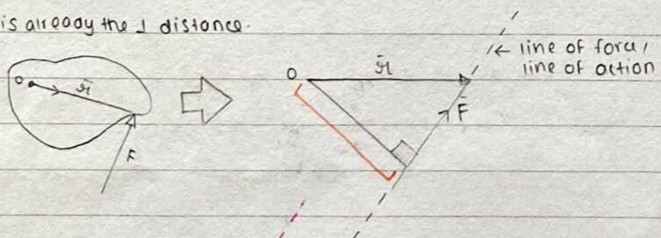
$\bar{\tau}$ = product of force and the ⊥ distance from the line of force to the point of rotation.

① 

② 

(i) Draw the line of force
(ii) Draw a line from the r's arrowhead to the line of force / the force → at 90° to the F.
This is the ⊥ distance.

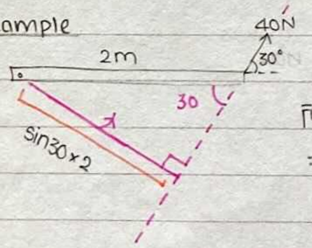
Here, r is already the ⊥ distance.

③ 

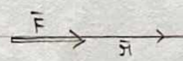
Angle btwn 2 vectors is always measured between the points of origin.

ⓧ 30 ⇒ 30 30 : θ ✓

example

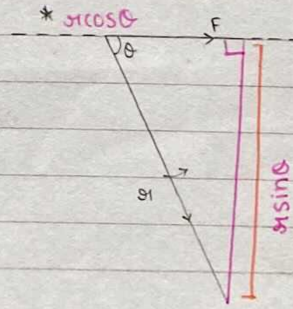


$\bar{\tau} = 40N \times \sin 30 \times 2m = 40N \times \frac{1}{2} \times 2m$
$= 40Nm$ ACW (outwards ⊙)



F is parallel to r,
so θ = 0.

$\bar{\tau} = F \times r\sin\theta = F \times 0 = 0$ No torque



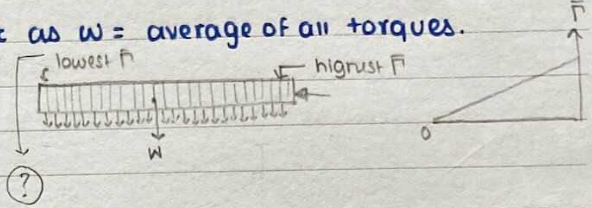
* r is going towards θ

$\bar{\tau} = F \times r\sin\theta$

3.02

CENTRE OF MASS

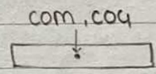
- mass of body is concentrated around this point
- entire weight assumed to be acting through centre point as W = average of all torques.
- point about which the moments of all the point masses balance.

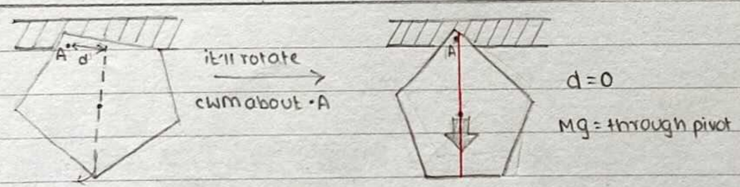


centre of mass (COM)
imaginary point of a given mass distribution, where the entire mass can be assumed to be concentrated at.

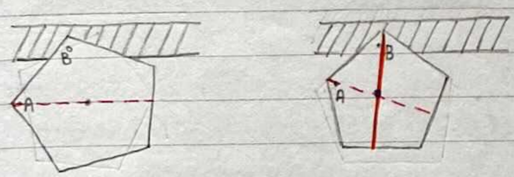
centre of gravity (COG)
imaginary point in a given mass distribution about which the moment of weight of all points is equal to 0.
imaginary point of a given weight distribution, where the entire weight can be assumed to be concentrated at.

→ In a uniform gravitational (g) field com and cog are one and the same point.
→ In a non-uniform g field com and cog are different.
→ In a gravity free space cog can't be defined / doesn't exist



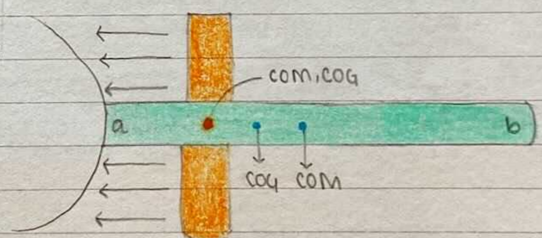


com lying on the vertical line passing through the pivot.
The exact position is still unknown, but the line it lies on is known.



Another pivot, ·B, was taken and the vertical line passing through it was found once the card came to rest.

The point where the first and the second lines intersect is the centre of mass.



: end b is pulled less than a as Earth's gravitational pull decreases in outer space.
Dist. btwn COG and b > dist btwn COG and a.

: all points equally close to Earth (approx) so they are equally pulled and COM = COG
equal gravitational force acting on all points

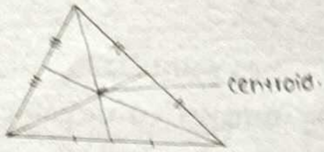
Location of C.O.G.

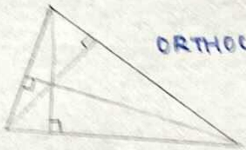
(i) Medium: a line joining a vertex to the point on the opposite side which bisects the opposite side.

(ii) Altitude: a line joining the vertex, which is ⊥ to the opposite side.

(iii) Angular bisector: a line joining the vertex bisecting the internal angle.

CENTROID - point of intersection of mediums



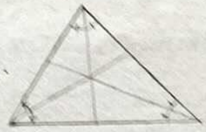


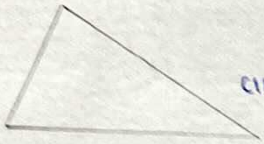
ORTHOCENTRE - point of intersection of altitudes.

incentre: equidistant from sides of the triangle (can draw a ○ around touching all sides) (⊥ dist from incentre to each side is the same.

circumcentre is equidistant from all vertices.

INCENTRE : point of intersection of angular bisectors.





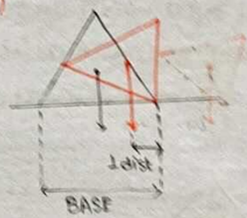
CIRCUMCENTRE: point of intersection of ⊥ bisectors

- C.O.M is located at the centroid of a triangle

Types of equilibrium

(i) Stable equilibrium : an object remains stable if the cog remains within the base of the object.

(i)



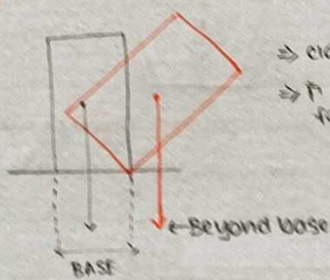
⇒ anticlockwise M in ▽

⇒ M restores object to its original position

(ii) unstable equilibrium

the cog is outside the base. The object falls once released.

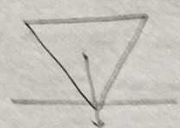
(ii)



⇒ clockwise M in ◇

⇒ M causes object to fall

(ii)



(iii) Neutral equilibrium

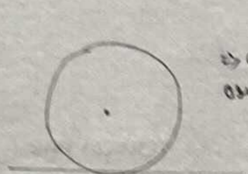
The C.O.M doesn't change with a small displacement

A cylinder:

(i) while falling is under unstable equilibrium

(ii) that has fallen is under neutral equilibrium

(iii)



⇒ C.O.M doesn't change with the orientation in a sphere convert mass from g to kg to calculate N.

☆ 250g = 0.25kg = 2.5 N

3.04

STRETCHING AND COMPRESSION

Hooke's Law

elasticity - the property of a body by which it has the ability to return to its original shape after going through / undergoing deformation. (extension or compression).

- an object gets deformed under the application of a force.

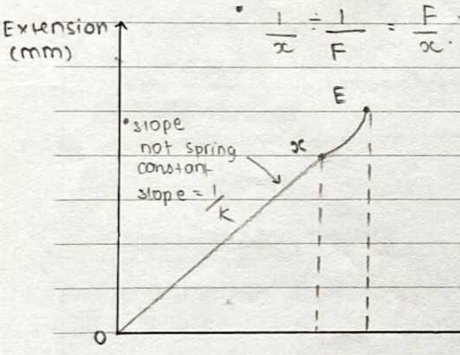
→ Natural length: original length, when no forces are acting F=0.

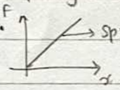
→ Elastic sub: subs which are able to regain their original shape back after the removal of the external force (a property : elastic). eg. spring.

→ Plastic sub: subs which get permanently deformed on the application of an external force.
— spelling —

→ extension in spring: final length - natural length.

• $\frac{1}{x} \div \frac{1}{F} = \frac{F}{x}$ = Spring constant





O - x : ① line is straight and passes through origin (L = 0, Ext = 0).

② Spring constant = $\frac{\text{Load}}{\text{Extension}}$ F = Kx (K = S constant)

x = limit of proportionality.

E : Elastic Limit. aka yield point.

As long as the graph obeys a straight line:

(i) Body is obeying Hooke's law

(ii) slope = spring constant. (Hookes constant)

(iii) Extension ∝ Force applied ($F \propto x$)

Extension ∝ Load

F = Force → Newton

x = extension → mm

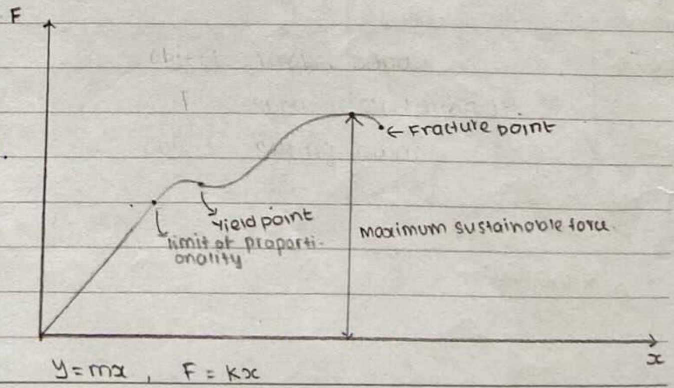
K = F/x → N/mm.

Load is the 'F'. It is measured in Newtons. weight ✓ mass ✗

Beyond x, until E:

Ratio of extension : Load ($\frac{\text{extension}}{\text{Load}}$) isn't constant.

However it is still elastic.



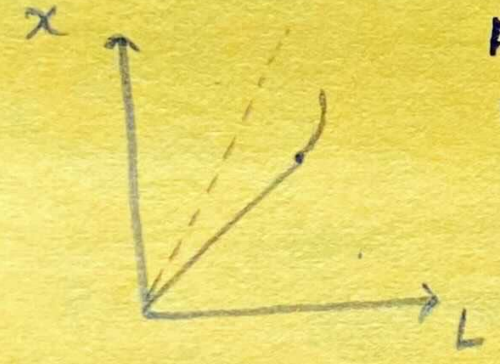
y = mx , F = Kx

The moment the object reaches the max. sustainable force (breaking force) it will continue extending and then break. (not elastic anymore).

Fracture point : Maximum length, at which it will break.

Yield point : Point beyond which the object behaves like a plastic (lost elasticity). Object behaves in a random way.

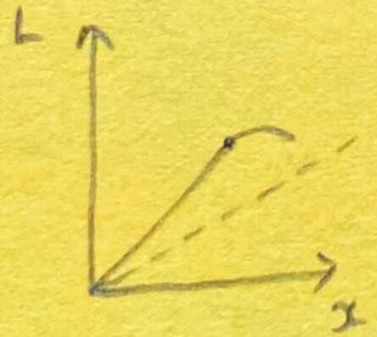
Fracture



A. curves upwards

B. a spring with a smaller k constant would have a steeper slope

$\text{slope} = \frac{1}{k}$ *



A. curves downwards

B. a spring with a smaller k would have a flatter slope

$\text{slope} = k$

A. shows that a small increment in load creates a large extension.

* slope → x / L

B. $k = \frac{F}{x} = \frac{L}{x}$ $x > L$.

$k = L/x$ $\frac{1}{k} = 1 \div L/x$

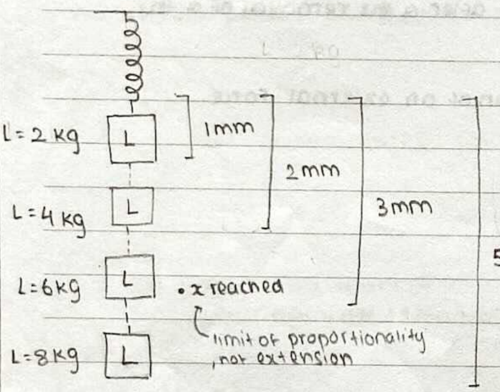
$= 1 \times \frac{x}{L} = x/L$

HOOKES LAW.

FORM 1 : The extension produced in a spring is directly proportional (α) to the amount of applied force within limits of elasticity. $F = kx$.

FORM 2 : within limits of elasticity the extension in directly proportional to the applied load.

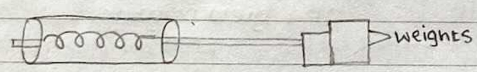
FORM 3 : within limits of elasticity, stress generated in a mechanical system is α to the strength applied.



$\frac{2kg}{1mm} = \frac{4kg}{2mm} = \frac{6kg}{3mm} \neq \frac{8kg}{5mm}$

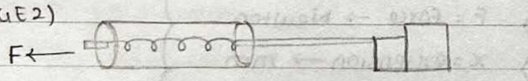
$\frac{20N}{1mm} = \frac{40N}{2mm} = \frac{60N}{3mm} \neq \frac{80N}{5mm}$ $(mg = m \times 10)$

STAGE 1)



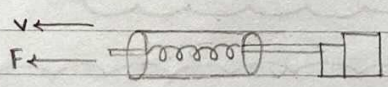
$F=0$ $V=0$ $x=0$

STAGE 2)

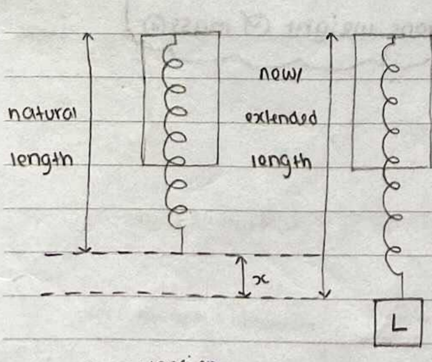


F: applied $v=0$ x = present.

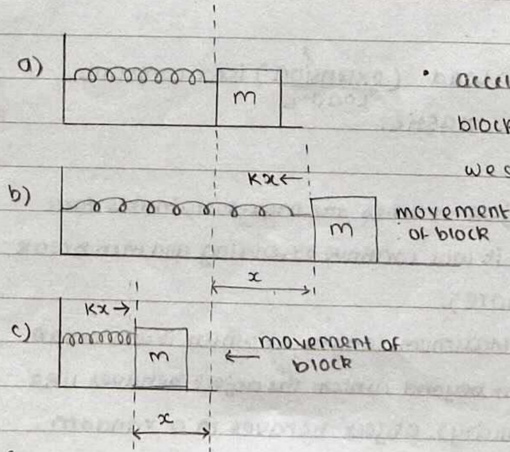
STAGE 3)



F = applied (same) v = present $x=0$.



x = extension



• acceleration of the block will be -ve but we show only magnitude +ve.

(kx opposes mvment)

(c) compressed to the same level b) is stretched = x)

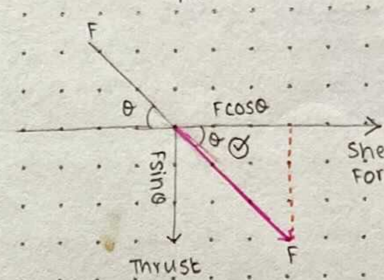
PRESSURE

(1) Solid pressure

$\dfrac{\perp F}{\text{Unit Area}}$

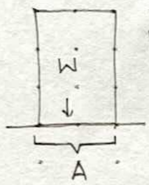
★ [$P = \dfrac{\text{Thrust}}{\text{Area}}$ Ratio of thrust applied to the area of application of thrust.

☑ θ is here (not the opposite side) as it is from the points of origin

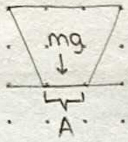


(1.) SHEER FORCE : any force that acts parallel to the surface
(2.) THRUST : acts ⊥ to the surface. (aka compressive force)

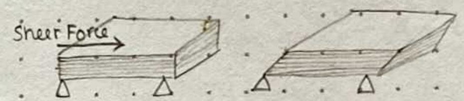
Thrust



$P = \dfrac{W}{A} = \dfrac{mg}{A}$



$P' = \dfrac{mg}{A'}$



Book form as parallelogram.

$\dfrac{P'}{P} = \dfrac{A}{A'}$ as $P \propto \dfrac{1}{A}$ (if $P\uparrow A\downarrow$)

• if weights (W) are equal, the $W = P \times A$ and $W = P'A'$
so $PA = P'A'$ so $\dfrac{P}{P'} = \dfrac{A'}{A} \Rightarrow \dfrac{P'}{P} = \dfrac{A}{A'}$

A body can be under 2 stresses at once:
(i) sheer stress
(ii) compressive force

• Pressure is inversely proportionate to area.

Sheer Force

★ [$\dfrac{\text{Sheer Force}}{\text{Area of contact}}$ = Sheer Stress

Snow: weak substance: flat skiis so as not to sink (snow yields less due to ↓P)
Ice: if P is applied the melting point reduces, so the blade of ice-skates increases P and the ice forms H_2O. The skates slip through easily. once the skates have left the H_2O freezes back to ice as the P has gone - for anamalous substances

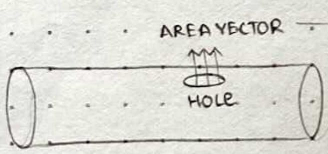
Properties of pressure

(i) it is a scalar quantity (P has no direction)
(ii) $P \cdot \bar{A} = \bar{F}$

$P = \dfrac{\text{magnitude of force}}{\text{magnitude of area}}$

$P = \dfrac{F\perp}{A}$ Both F & A are vectors

$P \cdot \bar{A} = \bar{F}$. $\bar{F}$ has to be a vector as $\bar{A}$ is a vector

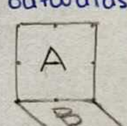


The H_2O shoots out perpendicularly. the direction is due to the area's direction, not pressure's (as P doesn't have a direction)

PIPE WITH H_2O AT ↑ PRESSURE

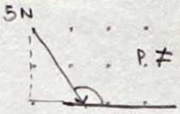
(iii) S.I unit: Pascal
N/m^2 = Pascal

Area vector is always orientated perpendicularly outwards.

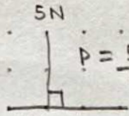


A and B's area are in different directions.

★ ★ ★ ★ ★ ★ Pressure is PERPENDICULAR force per unit area, not just any force.



$P \neq \dfrac{5}{10}$ Pa



$P = \dfrac{5}{10}$ Pa

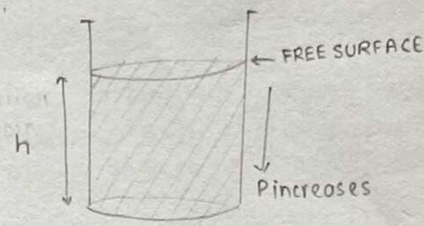
3.06

PRESSURE IN LIQUIDS

1) Liquid pressure

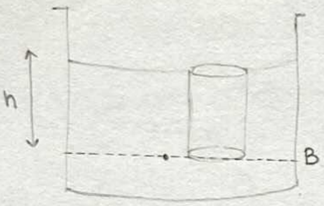
- exerted in / acts in all directions
- The liquid pushes on every surface in contact with it (no matter which way it is facing)
- Pressure increases with depth
- Pressure depends on (i) density of liquid (ii) vertical depth from the free surface.

It is independant of the container's shape or volume.
↓
Pressure at the points on the same horizontal line / level in a liquid is the same.



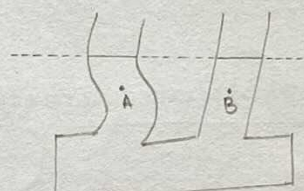
h: vertical depth from the free surface to any point in the liquid.
⇒ a liquid has only 1 free surface.

Calculating liquid Pressure



B: horizontal surface at depth h
▯ : has (i) an area (A) (ii) height (h) and volume (V)
V : Ah
ρ : Density of fluid
mass : V × D : $Ah \times \rho$
(W) weight : mg : $Ah\rho g$
Pressure : $\frac{Ah\rho g}{A} = \rho g h$.
↓
at the point (•)

[ASSUMPTION - NO EXTERNAL PRESSURE]

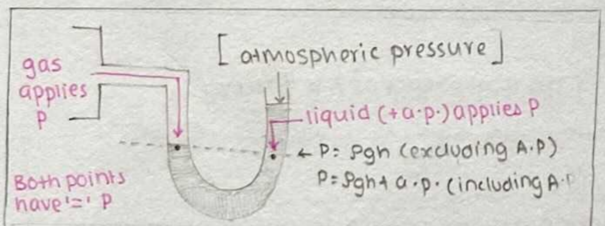


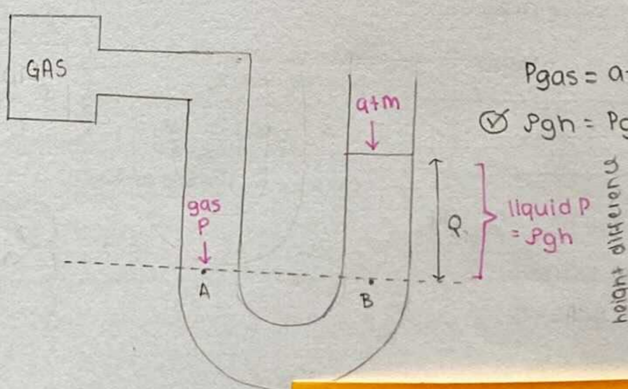
A and B have the same Pressure.

Manometer

used to measure pressure difference

[Assuming : The system is always at rest; the object is kept in an inertial frame; system doesn't accelerate
: Neglecting atmospheric pressure]



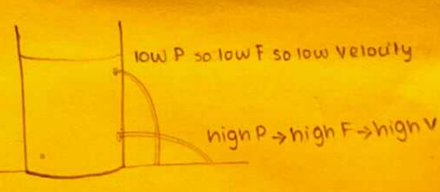


$P_{gas} = atm + \rho g h$

$\rho g h = P_{gas} - atm$

gauge pressure ⇒ $\rho \times g \times$ height difference between two mercury columns = $\rho g Q$

- The gas P = atm + liq. P : we know this as both the levels of liquid are stable. If, suppose, the gas P > liqP + atm then the column A would be pushed down more, resulting in the rise of column B. But there is no fall/rise in the liquid levels so the pressures balance.



How far the liq. travels actually depends on (i) time (ii) speed, but this is generally the case.

Pressure near the free surface is low as the vertical depth / height from the free surface is low.
P at the topmost point of a fluid = 0